# The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, RESITS/SUMMER 2020-2021

**DYNAMICS AND CONTROL**

Time allowed THREE Hours plus 30 minute upload period

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***Open-book take-home examination***

***Answer ALL questions***

*You must submit a single pdf document, produced in accordance with the guidelines provided on take-home examinations, that contains all of the work that you wish to have marked for this open-book examination. Your submission file should be named in the format '[Student ID]_MMME2046.pdf'.*

*Write your student ID number at the top of each page of your answers.*

*This work must be carried out and submitted as described on the Moodle page for this module. All work must be submitted via Moodle by the submission deadline.* ***Work submitted after the deadline will not be accepted without a valid EC.***

*No academic enquiries will be answered by staff and no amendments to papers will be issued during the examination. If you believe there is a misprint, note it in your submission but answer the question as written.*
*Contact your Module Teams Channel or SS-AssessEng-UPE@exmail.nottingham.ac.uk for support as indicated in your training.*

***Plagiarism, false authorship and collusion are serious academic offences*** *as defined in the University's Academic Misconduct Policy and will be dealt with in accordance with the University's Academic Misconduct Procedures. The work submitted by students must be their own and you must declare that you understand the meaning of academic misconduct and have not engaged in it during the production of your work.*

**ANSWER ALL QUESTIONS**

Machine Dynamics (30 points)

1. Figure Q1 shows a slider-crank mechanism in which a motor drives crank AB about a fixed axis at A causing connecting rod BC to exhibit general plane motion and the slider at C to translate in the vertical direction. The relevant dimensions of the links are shown in the figure. At the instant shown angle $\gamma = 13.63^\circ$, the crank AB has constant angular speed $\omega = 100\,\text{rpm}$, the connecting rod BC has angular velocity $\omega_{BC} = 2.5405\,\text{rad/s}$ and the slider has velocity $v = 0.9209\,\text{m/s}$.

   (a) Calculate the magnitude and direction of the angular acceleration of connecting rod BC and the acceleration of the slider at C. [18]

   (b) Draw and annotate Free Body Diagrams for the crank, connecting rod and slider. Note: you can assume that the pin joints are frictionless and the weight forces acting on the bodies can be neglected. [6]

   (c) If the masses of the piston and connecting rod are 1 kg and 2kg respectively and the moment of inertia of BC about B is 2 kgm$^2$, calculate the horizontal and vertical reaction forces acting at C. [6]

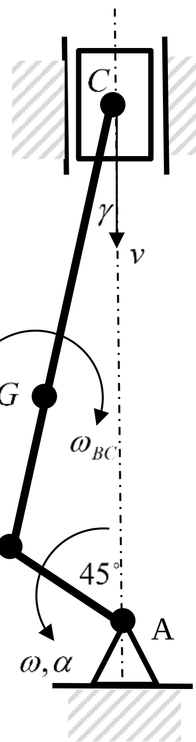


Figure Q1
AB = 100 mm, BC = 300 mm, BG = 100 mm.

Controls (30 points)

2. The block diagram for an open loop, 2nd order system is shown in Figure Q2.

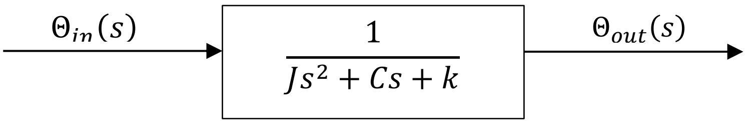


Figure Q2

(a) For $J$=2 kg·m and $k$=6 N·m/rad, calculate the value of $C$ (N·m·s/rad) that will result in a critically damped system. [5]

(b) Calculate the natural frequency of the system. [5]

(c) For a step input given by $\Theta_{in}(s) = {}^{2}/_{s}$, calculate the magnitude of the steady state error defined as:

$$E_{ss} = \lim_{t \to \infty} \left( \theta_{in}(t) - \theta_{out}(t) \right)$$

[5]

Hint:

| Function | Laplace Transform |
|---|---|
| $1 - \omega t e^{-\omega t}$ | $\dfrac{\omega^2}{s(s^2 + 2\omega s + \omega^2)}$ |

The open loop system in Figure Q2 is then incorporated into a closed loop system as shown in Figure Q3.

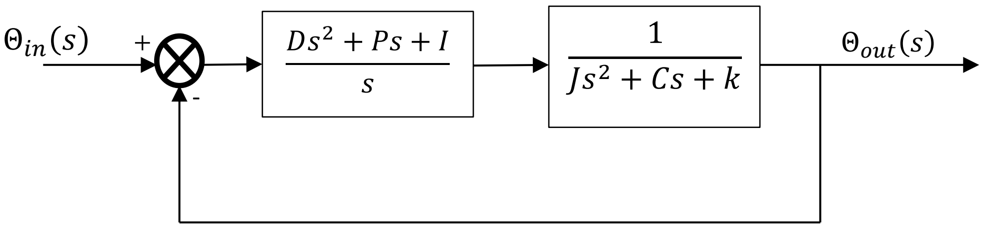


Figure Q3

(d) What is the overall transfer function, $G(s) = \frac{\Theta_{out}(s)}{\Theta_{in}(s)}$ for the system shown in Figure Q3? [5]

(e) For the values of $P$, $D$, $I$, $J$ and $k$ given in Table 1, calculate the range of values of $C$ for which the system will remain stable. [10]

| *P* | *D* | *I* | *J (kg)* | *K (N/m)* |
|---|---|---|---|---|
| 2 | 1 | 10 | 2 | 6 |

Table 1

Vibrations (25 points for Q3 and 15 points for Q4; Total 40 points)

3. Figure Q3 shows a rigid bar $AB$ which pivots about fixed point $O$.

DATA
$I_O$ = 0.5 kg m$^2$
$L_1$ = 0.4 m
$L_2$ = 0.6 m
$K_1$ = 1000 N/m
$K_2$ = 1000 N/m
$c$ = 50 Ns/m

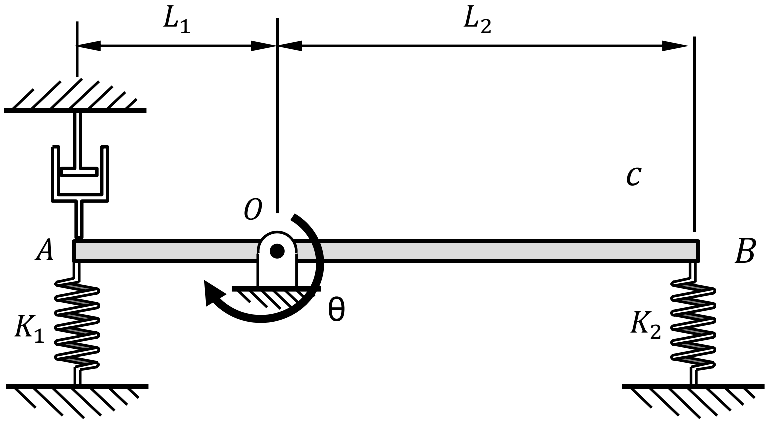


Figure Q3

(a) Draw a fully annotated Free Body Diagram for the bar. [4]

(b) Derive the equation of motion governing rotation of the beam θ(t) about pivot O. [3]

(c) Calculate the undamped natural frequency $\omega_n$ and damping ratio $\zeta$ for the system. [4]

(d) The end B is lifted up by 0.05 m and the bar is then released from rest. Determine the resulting transient angular displacement at O as a function of time, $\theta_{tr}(t)$. [6]

(e) What frequency will this system vibrate at after being released from rest? [2]

(f) For the subsequent vibration, calculate the speed of end B when it first passes through the equilibrium position. [6]

4. Figure Q4 shows a uniform beam of length, $L$, density $\rho$, Young's modulus of elasticity $E$, and second moment of area $I$. One end of the beam is clamped, while the other end is connected to a spring of stiffness, $k$, as shown.

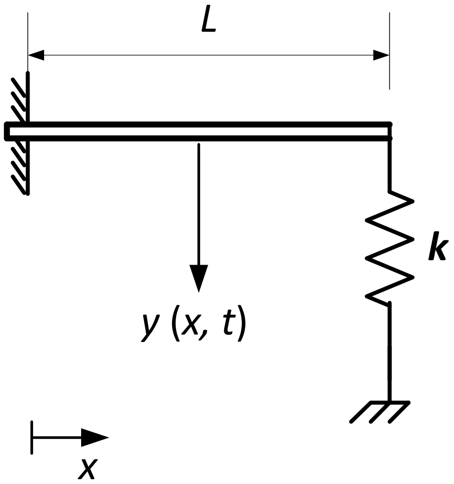


Figure Q4

(a) Using boundary conditions determine the generalized matrix, $[Z]\{C\}=\{0\}$, that could be used to solve for undamped natural frequencies and mode shapes of the beam. Note: It is not required for you to solve for the constants $\{C\}$, only show the generalized matrix and terms contained in $[Z]$. [13]

(b) Provide simple sketches for the first two modes of vibration. [2]